MODEL FOR ESTIMATING TEMPERATURES IN CONCRETE PAVEMENTS

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ABSTRACT: Concrete pavement design and structural evaluation are connected to the pavement strain-stress condition during life-service. Strains and stresses are due to traffic load and climatic conditions. This paper deals with a closed-form thermal model to estimate hour by hour the temperature of a concrete slab resting on a Winkler-type elastic foundation and the analysis of the effect of temperature distribution on thermal stresses in concrete pavement. The temperature gradient across the concrete slab thickness causes horizontal and vertical displacements as well as warping stresses in the concrete pavement slabs. Pavement temperatures affect concrete stresses and movement of cracks and transverse joints, so thermal conditions are an important factor in global structural phenomena in a concrete slab. The calculated values have been validated by comparison to air and pavement temperature measurements registered in Italian airport locations. Results show that this model describes the effects of climatic factors accurately and is well suited to use in further studies.

KEY WORDS: Temperature, model, concrete, pavement, heat, slab

1. INTRODUCTION

Thermal conditions in concrete slabs affect strains and stresses [1,2]. So ignoring them exposes the concrete pavements to unacceptable risk of premature failure [3]; stresses imposed by temperature must be considered as important as stresses inducted by traffic [4,5]. With this aim, there is the need to describe temperature at any depth by knowing input data that affect temperature fluctuation in pavements. This paper presents a closed-form thermal model to estimate hour by hour the temperature of a concrete slab resting on a Winkler-type elastic foundation and the analysis of the effect of temperature distribution on thermal stresses in concrete pavement. As described in literature, air temperature, wind speed, solar radiation, physical and thermal properties of materials influence the pavement temperatures in daily and seasonal cycles.

It is known that thermal exchanges between air and solid body occurs by radiation and convection [6,7]. Thermal radiation can be described according to the Stephan–Boltzmann law:

\[ U = \sigma \cdot T^4 \] (1)

\( U \) total energy radiated per unit surface area of a black body
\( T \) thermodynamic temperature
\( \sigma \) Stefan–Boltzmann constant.

Heat transfer per unit surface through convection was first described by Newton: the relation is known as the Newton’s law of cooling. The natural or free convection can be described according to the equation:

\[ q = h \cdot A \cdot dT \] (2)

\( q \) heat transferred per unit time
\( h \) convective heat transfer coefficient of the process
\( A \) heat transfer area of the surface
\( dT \) temperature difference between surface and the bulk fluid.
The convective heat transfer coefficient \( h \) comes from correlations of dimensionless numbers, typically they are: the Nusselt number, the Prandtl number, the Grashof number. So, \( h \) depends on the flow properties and the geometrical, physical and thermal characteristics of problem.

Assuming no internal heat generation and constant thermal characteristics of material, at the boundary, the difference between the incoming heat flux and the outgoing heat flux is equal to the penetrating heat flux in the body:

\[
q_{\text{incoming}} - q_{\text{outgoing}} = q_{\text{penetrating}} = q \tag{3}
\]

Inside the pavement, conduction’s heat exchange is given by the Fourier law for isotropic and homogeneous solid materials:

\[
q = -k \cdot \text{grad}(T) \tag{4}
\]

So, heat transfer in one dimension is regulated by Equation 5:

\[
\frac{dT}{dt} = d \cdot \frac{\partial^2 T}{\partial z^2} \tag{5}
\]

The thermal diffusivity is equal to:

\[
d = \frac{\kappa}{\rho \cdot c} \tag{6}
\]

All heat exchange process depends upon the amount of radiation. At the Earth’s surface, quantities of solar radiation undergo daily cycles of change. Variations in radiation are primarily responsible for the fluctuating air temperature over a 24 hour period. The following graph shows ideal average curve of radiation for a typical site based location at 40° of latitude on summer sunny day. For all dates, and so for all sunny days, peak of radiation occurs at solar noon, when the sun attains its greatest height above the horizon.
2. ANALYTICAL SOLUTIONS

Regarding the complexity of this problem, Barber [7] defined a sinusoidal function to describe the solution of the one-dimensional problem of heat transfer across a semi-infinite, isotropic, homogeneous solid surface which is in a steady regime, although periodic, with a boundary sinusoidal temperature variation forced in the surface (Equation 7):

$$T(z, t) = T_{ag} + \Delta T \cdot \sin \left(\frac{2\pi \cdot t}{\tau} - z \frac{\pi}{d \cdot \tau}\right)$$  \hspace{1cm} (7)

- $T_{ag}$: average daily air temperature
- $\Delta T(z)$: differential thermal amplitude at $z$ depth
- $t$: time in hour
- $z$: pavement depth
- $\tau$: period of temperature variation

Furthermore, for semi-infinite medium, the theoretical phase displacement $\varphi$ and damping values $\gamma$ are shown Equation 8 and Equation 9 respectively:

$$\varphi = \frac{k \cdot z \cdot \tau}{2\pi}$$  \hspace{1cm} (8)

$$\gamma = \sqrt{\frac{\pi}{a \cdot \tau}}$$  \hspace{1cm} (9)

Equations below show solutions of Barber’s theory in Equation 10 [7] and Thomlinson’s theory in Equation 16 [8]:

$$T(z, t) = T_{ag} + R + \left(\frac{A_{ag}}{2} + 3R\right) \cdot F \cdot e^{-\frac{Cz}{2}} \cdot \sin \left(0.262t - Cz - \arctan \frac{C}{C + H}\right)$$  \hspace{1cm} (10)
\( T(z,t) \)  
Temperature of pavement at depth \( z \) at hour \( t \)

\( T_{ag} \)  
Average daily air temperature

\( A_g \)  
Daily range in air temperature

\( R \)  
Calculated daily solar radiation contribution to air temperature

\[
R = \frac{2 \cdot b \cdot R_s}{3 \cdot 24 \cdot h} \tag{11}
\]

\( b \)  
Surface absorptivity to the total solar radiation

\( R_s \)  
Daily solar radiation

\[
h = (1.3 + 0.62 \cdot v^{0.75}) \tag{12}
\]

\( v \)  
Average wind speed

\( h \)  
Heat transfer coefficient

\[
F = \frac{H}{\sqrt{(H + C)^2 + C^2}} \tag{13}
\]

\[
H = \frac{h}{k} \tag{14}
\]

\[
C = \frac{0.131}{d} \tag{15}
\]

Where \( d \) is the thermal diffusivity of concrete.

\[
T(z,t) = T_{sg} + T_o \cdot e^{-\frac{x}{\sqrt{t_c \cdot d}}} \cdot \sin\left(\frac{2\pi t}{t_c} - \sqrt{\frac{\pi}{t_c \cdot d}} z\right) \tag{16}
\]

\( T_{sg} \)  
Average daily air temperature

\( T_{sg} \)  
Average daily surface temperature

\( T_0 \)  
Daily range in surface temperature

\( t_c \)  
Daily period

So, most known thermal models describe temperature distribution as a sinusoidal function. After calculating maximum and minimum temperatures with Barber’s model, we applied these thermal values to Thomlinson’s model to compare results. Figure 2 shows the two curves almost overlapping.
Nevertheless, real development of pavement temperature during a cloudless day is not sinusoidal, because ground solar radiation changes during the daytime [9]. Experimental measurements show the maximum level of solar radiation is when the sun is at zenith and also curves of the pavement surface temperature show the qualitative development of surface temperature similar to that of solar radiation. The phase shift and the delay of pavement surface temperature is related to the thermal characteristics of concrete and the circadian temperature cycle.

Faraggi, Jofre and Kraemer [10] modified Thomlinson’s model and built a new mathematical thermal theory defining a double sinusoidal thermal law of daily oscillation of temperature in concrete pavements. Unlike the two theories presented above, the law of variation of the temperature depends on the hour of day when the thermal state is calculated: Equation 17 is valid between sunrise and zenith, Equation 18 is valid between zenith and sunrise.

\[
T(z,t) = T_{sg} \cdot \beta \cdot \gamma + A_s \cdot \beta \cdot \gamma \cdot (1 - \alpha) \cdot e^{-\frac{\pi}{d \cdot S_{sa}}} \cdot \sin \left( \frac{2t - S_h}{2S_n} \cdot \pi - \frac{\pi}{d \cdot S_{zh}} \right) \tag{17}
\]

\[
T(z,t) = T_{sg} + A_s \cdot e^{-\frac{\pi}{d \cdot S_{sa}}} \cdot \sin \left( \frac{4(t + S_a) - S_a}{2S_a} \cdot \pi - \frac{\pi}{d \cdot S_{za}} \right) \tag{18}
\]

- \(T_{sg}\) average daily surface temperature
- \(A_s\) daily surface oscillation amplitude temperature
- \(S_h\) number of hours between sunrise and zenith
- \(S_n\) number of hours between sunset and the next sunrise

\(S_h\) and \(S_n\) values must satisfy the condition:

\[
2S_h + S_n = 24 \tag{19}
\]

In the curve, \(t = 0\) coincides with the sunrise hour.

\[
S_a = 2 \cdot (S_h + S_n) \tag{20}
\]
\[ S_{za} = \frac{z^2 S^2}{d \cdot \pi \cdot u_z} \]  
\[ S_{zh} = \frac{z^2 S^2}{d \cdot \pi \cdot u_z} \]  
\[ u_z = \frac{z}{2} \sqrt{\frac{t_c}{d \cdot \pi}} \]  

The Faraggi et al. method introduced three coefficients for the theoretical formulas in order to align the model results with those of the experimental data:

- \( \alpha \) adjustment coefficient;
- \( \beta \) corrective wind coefficient;
- \( \gamma \) corrective rain coefficient.

The model is valid for the Spanish climatic zones. Solutions proposed by Equation 17 and Equation 18 are interesting because the temperature model produced accurate predictions of qualitative trend of surface concrete pavement temperature during daylight hours in cloudless daytime, as represented in [10] and shown in Figure 3; time 0 starts from sunrise. The temperature curve in Figure 3 has qualitative development similar to real curves measured during a cloudless day and represented in literature [10,11,12].

![Figure 3. Development of pavement surface temperature using Faraggi et al. model](image-url)

3. METHOD

The exposed mathematical models are used here to predict temperature of Italian concrete pavements. In situ pavement measurements have been used to calculate real magnitude and variations in the temperature gradient and validate a new prediction model.

Available experimental data show three items omitted by the previous models:

1. real daily temperature distribution doesn’t have a perfect sinusoidal variation. Oscillation is not symmetric with respect to the average temperature value;
2. in the hours between sunrise and zenith, temperature grows more swiftly than it decreases in the hours between zenith and sunrise. So, the solution proposed by Faraggi et al. may be more realistic than Barber’s model;
3. Experimental data show that hours of minimum and maximum air temperature values have delay with respect to sunrise and sunset hours.

To consider the first and second items, the model proposed by Faraggi et al. has been integrated with Barber’s model [13]. New thermal law calculates daily temperature range by the first model, which returns a closed form solution, and defines sinusoidal trend by Faraggi et al. model. Equation 2 is valid between sunrise and zenith, Equation 2 is valid between zenith and sunset.

\[ T(z, t) = T_{ag} + R + \left( \frac{A_g}{2} + 3R \right) \cdot F \cdot e^{-cz} \cdot \sin \left( \frac{2t - S_h}{2S_h} \cdot \pi - z \cdot \sqrt{\frac{\pi \cdot \gamma \cdot c_s}{24 \cdot k}} \right) \]  

\[ T(z, t) = T_{ag} + R + \left( \frac{A_g}{2} + 3R \right) \cdot F \cdot e^{-cz} \cdot \sin \left( \frac{4(t + S_n) - S_z}{2S_z} \cdot \pi - z \cdot \sqrt{\frac{\pi \cdot \gamma \cdot c_s}{24 \cdot k}} \right) \]  

To consider the third item, it needs to define two corrective coefficients which depend on the season and the thermal characteristics of concrete. Present research has not yet defined the delay coefficients, so experimental available data are used to set delay times.

4. VALIDATION ACTIVITIES AND RESULTS

Air and pavement temperatures have been collected during Falling Weight Deflectometer tests and specific measures of temperature. Only airport reliefs were available because of the lack of road concrete pavements in Italy. The concrete temperatures measured in the field were compared to the predicted values to validate the temperature model.

More than 1500 measurements from two locations in the northern and central part of Italy have been examined. All collected air temperatures have been compared to historical temperature data collected by the official observing stations located in those airports where air and pavement temperatures were measured. Available temperature data have been grouped by measurement days, in both winter and summer seasons during three weeks. Measurements have been collected both in daylight and nighttime hours, however the available nighttime measurements are much more numerous than daylight measures.

To distinguish genuine temperature measurements from incorrect ones, data that were beyond expected climatic values have been removed (for example, measurements inconsistent with the expected performance of the temperature or abnormal values). Then, before validating the new thermal curve, the data set was smoothed with a Simple Moving Average. The obtained patterns have been used to compare measured temperatures to predicted ones. The historical data of weather collected during FWD test days allowed to calculate predicted temperatures using the above proposed model. The physical and thermal properties for concrete pavements, assumed equal to typical literature values, are listed below:

<table>
<thead>
<tr>
<th>Property of concrete</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volumetric mass</td>
<td>2300</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>1.2</td>
<td>W/m K</td>
</tr>
<tr>
<td>Thermal diffusivity</td>
<td>0.6 E-06</td>
<td>m²/h</td>
</tr>
<tr>
<td>Specific heat</td>
<td>0.9</td>
<td>kJ/kg K</td>
</tr>
<tr>
<td>Absorption coefficient</td>
<td>0.6</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 4 compares thermal curves. It represents measured and calculated nighttime surface temperature in central Italy during summer season. This graphical representation highlights a good fit of proposed curve compared to measurements.
To analyze the quality of the fit of the theory to the experimental temperatures, quantitative techniques of model validation have been adopted. For each day, for each season and for each site, the residuals \( r_i \), differences between the observed temperatures \( O_i \) and the corresponding predicted ones \( P_i \),

\[
r_i = O_i - P_i
\]  

have been used for computing the mean residual (MR) and the root-mean-square deviation (RMSD).

In Table 2 are listed results referred to central Italy.

<table>
<thead>
<tr>
<th></th>
<th>Central Italy</th>
<th>Northern Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Season</strong></td>
<td><strong>Unit</strong></td>
<td><strong>Summer</strong></td>
</tr>
<tr>
<td>MR °C</td>
<td>0.22</td>
<td>0.10</td>
</tr>
<tr>
<td>RMSD °C</td>
<td>0.23</td>
<td>0.17</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

The present paper suggests a innovative analytical method developed to estimate temperature in concrete pavements. In the model, solar radiation, slab dimensions, slab elastic modulus, modulus of subgrade reaction, wind speed and thermal characteristics of concrete are considered to provide accurate estimates of temperature. Data collected during FWD tests, conducted in two Italian airports, have been compared with analytical results obtained using the new proposed method. The analytical approach describes the phenomenon with an adequate approximation, as demonstrated by validation activities. The obtained temperatures give reasonable profiles in the concrete slab. The new method is valid to calculate temperature of concrete pavements built in Italy, but research is still in progress to examine more sets of temperatures and define corrective coefficients calibrated on the local weather conditions. The new model has been used to assess the temperature effects on the concrete pavements, so it has been implemented in a versatile, user-friendly Microsoft Excel-based computer program developed to verify and evaluate structural and economic performance of a concrete pavement [13,14].
AKNOWLEDGMENT: Authors gratefully acknowledge Nicola Fiore and Alessandro Marradi for their support and availability to provide thermal data.

REFERENCES:


